TRINITY COLLEGE

Semester Two Examination, 2023

Question/Answer booklet

MATHEMATICS METHODS UNITS 3&4

Section Two: Calculator-assumed

WA student number: In figures

In words

Your name

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

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Section Two: Calculator-assumed   **65% (98 Marks)**

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8   **(8 marks)**

The manager of a high street bank wants to know what proportion of their customers never use cash and has asked an employee to collect sample data by standing in the bank foyer for two hours before lunch on a Thursday morning and questioning as many people as they can.

(a)   Identify and explain two possible sources of bias with this sampling procedure.   (4 marks)

(b)   Briefly describe a sampling procedure that the manager could use in order to minimise all sources of bias.   (2 marks)

From the 225 responses obtained using a reliable sampling procedure, the manager was presented with the confidence interval $(0.1924, 0.2876)$ for the proportion of their customers who never use cash.

(c)   Determine the number of customers in the sample who said they never use cash.
                                                                               (2 marks)

Question 9   **(7 marks)**

The air pressure, P kPa, inside the tyre of a motor vehicle t seconds after it was punctured can be modelled by the equation $P = a + 122e^{kt}$, where a and k are constants.

The initial pressure in the tyre was 220 kPa and after 8.5 seconds it had dropped to 142 kPa.

(a)   Determine the value of a and the value of k.   (3 marks)

(b)   Determine

      (i)   the pressure in the tyre after 5 seconds.   (1 mark)

      (ii)   the time taken for the pressure in the tyre to fall to 99 kPa.   (1 mark)

(c)   Given that the pressure was falling at a rate of 8 kPa per second after 5 seconds, use the increments formula to estimate the pressure in the tyre after 5.1 seconds.   (2 marks)

Question 10   **(8 marks)**

Liu has a hen that on some days lays an egg and on other days it doesn't. The random variable X is the number of eggs that this hen lays in a day, so that

$$P(X = x) = \begin{cases} a(3x + 2) & x = 0, 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine the value of the constant a and hence state the probability that Liu's hen does not lay an egg in a day.   (2 marks)

Liu also sells jars of honey from a roadside honesty box in which she places 4 jars every morning. The random variable Y is the number of jars of honey Liu sells in a day so that

$$P(Y = y) = \begin{cases} k(y - 1.5)^2 & y = 0, 1, 2, 3, 4 \\ 0 & \text{otherwise} \end{cases}$$

(b) Determine the exact value of the constant k.   (2 marks)

Assume that X and Y are independent and that Liu has a good day when her hen lays an egg and she sells at least 2 jars of honey.

(c) Determine the probability that Liu has a good day.   (2 marks)

(d) Determine the probability that Liu has exactly 6 good days in a week.   (2 marks)
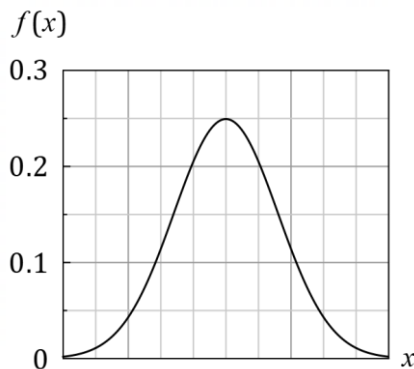
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Question 11   **(6 marks)**

(a) The probability density function f of a normal distribution with mean μ and standard deviation σ is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

The graph of a normal distribution with a mean of 18 is shown below. Estimate the standard deviation of the distribution. Justify your answer.   (2 marks)

(b) The weights, W grams, of eggs produced by the free-range hens at a small farm are normally distributed with a mean of 58.6 g and a standard deviation of 5.8 g.

Determine the probability that a randomly selected egg from this farm

(i) weighs more than 65 g.   (1 mark)

(ii) weighs no more than 55 g.   (1 mark)

(iii) weighs less than 55 g, given that it weighs less than 65 g.   (2 marks)

Question 12   **(10 marks)**

In the Shire of Murchison, 65% of the feral goat population is female.

(a)   Use a discrete probability distribution to determine the probability that when a random sample of 50 feral goats is taken, no more than 60% of them will be female.   (3 marks)

A large batch of simulations are run in which a random sample of 50 feral goats is selected from the Murchison population and the proportion of females in the sample calculated.

(b)   Describe the continuous probability distribution that the sample proportions from the simulations will approximate.   (2 marks)

Another simulation is run to obtain one more sample proportion.

(c)   Use the distribution from part (b) to determine:

     (i)   the probability that the sample proportion will be no more than 60%.   (1 mark)

     (ii)   the value of the constant k, given that there is an 90% chance that the sample proportion lies between 0.55 and k.   (2 marks)

It is thought that for older (over the age of 11) feral goats from this region, that 88% are female.

A second batch of simulations are run in which a random sample of 25 older feral goats is selected from the Murchison population and the proportion of females in the sample calculated.

(d)   State two reasons that the distribution of the sample proportions from this second batch will not yield such a good approximation to the type of distribution described in part (b) when compared to the first batch.   (2 marks)

Question 13 **(10 marks)**

Members of a toy library may take home up to 5 toys per visit. The following frequency table shows the number of toys borrowed by a random sample of 80 members.

Toys borrowed	0	1	2	3	4	5
Frequency	0	10	26	28	12	4

You may assume that relative frequencies obtained from the above data are reliable point estimates of probabilities and that the number of toys borrowed by any two members are independent.

(a) Determine the probability that a member borrows fewer than 5 toys, given that they borrowed at least 3 toys. (2 marks)

(b) Determine the probability that at least 3 of the next 5 borrowers take home an even number of toys. (3 marks)

(c) Show that the mean of the random variable X, the number of toys borrowed by a member, is 2.675 and determine the variance of X. (3 marks)

Observations indicate that members spend 7 minutes at the library plus 3 minutes per toy chosen.
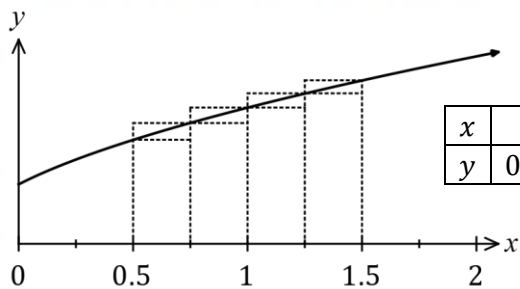
(d)   Determine the mean and standard deviation of the random variable T, the time in minutes spent by members at the toy library.   (2 marks)

Question 14 (8 marks)

The graph of $y = f(x)$ and a table of values for the function f are shown below.

x	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
y	0.96	1.38	1.68	1.94	2.16	2.42	2.64	2.84	3.03

(a) By considering the areas of the rectangles shown, demonstrate and explain why 2.17 is a reasonable estimate for $\int_{0.5}^{1.5} f(x)\, dx$. (3 marks)

(b) Determine, with justification, estimates for

(i) $\int_{0.5}^{1.5} 4f(x)\, dx$. (1 mark)

(ii) $\int_{0.5}^{1.5} f(x) + 4\, dx$. (2 marks)

(iii) $\int_{0}^{1} f\left(\frac{x}{2}\right) dx$. (2 marks)

See next page

Question 15 **(8 marks)**

Repair tasks undertaken by technical staff who work at an IT company are assigned minor, major or critical status. Over the long term, 4% of the tasks have been critical, 24% major and the remainder minor.

(a) Assuming that the long-term proportions are correct, determine the smallest sample size required so that the width of a 90% confidence interval for the proportion of minor tasks is less than 0.093. (3 marks)

At the end of one month, a manager suspects that the proportion of major tasks has changed and so she takes a random sample of 250 tasks from the last month, of which 45 were major.

(b) Use this sample to construct a 95% confidence interval for the proportion of major tasks. (3 marks)

(c) Does your confidence interval in part (b) support the managers suspicions? Justify your answer. (2 marks)

Question 16 (8 marks)

Alan works from home every Tuesday and always starts work after his digital clock first shows $8\!:\!10$ am and before it shows $8\!:\!30$ am.

The probability density function for T, the time in minutes after $8\!:\!10$ that he starts work, is $f(t)$ and is displayed at right.
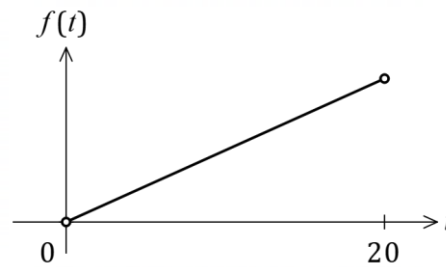
(a) Write the defining rule for the probability density function $f(t)$. (2 marks)

(b) Determine the probability that on a randomly chosen Tuesday, Alan starts work after his clock first shows $8\!:\!23$ am. (2 marks)

(c) Determine the mean and standard deviation of T. (4 marks)

Question 17   **(8 marks)**

The acceleration a m/s² of a train moving in a straight line at time t seconds is given by

$$a = k + \frac{1}{4}\cos\left(\frac{\pi t}{10}\right), \qquad 0 \leq t \leq 60.$$

Initially, the train was at an origin O and moving with a velocity of 4 m/s.

(a) Determine the velocity of the train after 10 seconds when the constant $k = 1.5$.   (3 marks)

After 20 seconds the displacement of the train relative to the origin O was 200 m.

(b) Determine the value of the constant k and hence calculate, to the nearest metre, the displacement of the train after 35 seconds.   (5 marks)
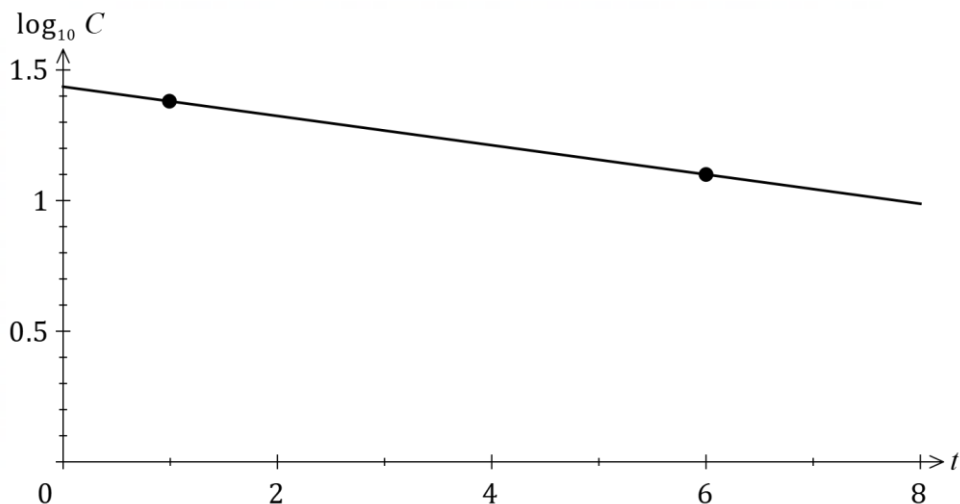
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Question 18   **(10 marks)**

The concentration C μg/mL of a pain-killing drug in a patient was observed for several hours.

The graph of $\log_{10} C$ against time t hours is linear and is shown below, passing through the points $(1, 1.38)$ and $(6, 1.1)$.

The relationship between C and t can be written in the form $\log_{10} C = at + b$.

(a)   Determine the value of the constant a and the value of the constant b.   (2 marks)

(b)   Determine where the straight line intercepts the horizontal axis and interpret what this point represents in the context of this question.   (3 marks)

(c)   Show that C can be expressed as a function of t in the form $C = k(10)^{at}$ and state the value of the constant k.   (3 marks)

(d)   Determine

       (i)   the concentration of the drug in the patient after 9 hours.   (1 mark)

       (ii)   the time taken for the concentration of the drug to first fall below 5% of its initial value.   (1 mark)
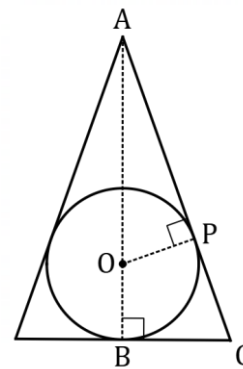
Question 19   **(7 marks)**

A right-cone of radius $BC = r$ cm, height $AB = h$ cm and volume V cm^3 is circumscribed about a sphere of radius $OB = OP = 9$ cm.

The diagram shows a cross-section through the centre of the cone and sphere.

Note that $\triangle ABC \sim \triangle APO$.

(a)   Show that $V = \dfrac{27\pi h^2}{h - 18}$.

(3 marks)

(b)   Use calculus to determine the minimum possible volume of the right-cone.   (4 marks)

End of questions

Supplementary page

Question number: _____